Universität Basel

Research Project

Diophantine Problems, o-Minimality, and Heights

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Project title Diophantine Problems, o-Minimality, and Heights
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The study of diophantine equations is one of the oldest parts of number theory. Its ultimate goal is to describe integer or rational solutions of polynomial equations. For polynomials in two variables, major results are: Siegel's Theorem on integral points on curves and Faltings' resolution of the Mordell Conjecture. They imply, under natural hypotheses, that a polynomial equation in two variables has only finitely many integral and rational solutions, respectively. In a greater number of variables, finiteness statements are known by work of Vojta, Faltings, and others in the context of the Mordell-Lang Conjecture when working inside an abelian variety. Roughly a decade ago, Zilber and Pink independently stated conjectures on unlikely intersections. Here points of arithmetic interest, e.g. those in the Mordell-Lang Conjecture, appear as intersections that are improbable for geometric reasons. Pink's version contains the André-Oort Conjecture on the distribution of CM or special points on certain moduli spaces. A strategy developed by Zannier has been one driving force towards these general conjectures. Central to this approach is a counting result by Pila and Wilkie in an o-minimal structure, a concept that originated in and has connections to model theory in mathematical logic. Part of this proposal aims to study new cases of the Zilber and Pink Conjecture by combining Vojta's approach to the Mordell Conjecture with the counting strategy. We also plan to investigate unlikely intersections in non-reductive groups and shed new light on connections between Siegel's Theorem and CM points.

Keywords families of abelian varieties, diophantine geometry, CM points, Mordell-Lang Conjecture, integral points, model theory, heights, Zilber-Pink Conjecture, special points, o-minimal structures, unlikely intersections, isogenies, Faltings height
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