Approximation of bi-variate functions : singular value decomposition versus sparse grids

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We compare the cost complexities of two approximationăschemes for functions $f\in H^p(\Omega_1\times \Omega_2)$ăwhich live on the product domain $\Omega_1\times\Omega_2$ăof sufficiently smooth domains $\Omega_1\subset\mathbb{R}^{n_1}$ăand $\Omega_2\subset\mathbb{R}^{n_2}$, namely theăsingular value / Karhunen-L\'oeve decomposition and the sparse grid representation. Here we assume thatăsuitable finite element methods with associatedă{\em fixed} order $r$of accuracy are given on theădomains $\Omega_1$and Ω_2. Then, theăsparse grid approximation essentially needs onlyă$\mathcal{O}(\varepsilon^{-q})$with $q=\frac{\max\{n_1,n_2\}}{r}$ăunknowns to reach a prescribed accuracyăεprovided that the smoothness of făsatisfies $p\ge r\frac{n_1+n_2}{\max\{n_1,n_2\}}$,ăwhich is an almost optimal rate. The singular valueădecomposition produces this rate only if $f$isăanalytical since otherwise the decay of theăsingular values is not fast enough. If $p\frac{n_1+n_2}{\max\{n_1,n_2\}}$, then the sparse grid approach gives essentially the rate $\mathcal{O}(\varepsilon^{-q})$with $q=\frac{n_1+n_2}{p}$while, for the singular value decomposition, we can only prove the rate $\mathcal{O}(\varepsilon^{-q})$with $q=\frac{2\min\{r,p\}\min\{n_1,n_2\}+2p\max\{n_1,n_2\}}{(2p-\min\{n_1,n_2\})\min\{r,p\}}$. We derive the resulting complexities, compare the two approaches and present numerical results which demonstrate that these rates are also achieved in numerical practice.

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